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Control Laws for Minimum Orbital Change—The Satellite Retrieval Problem

Timothy Cichan,* Robert G. Melton,† and David B. Spencer‡
Pennsylvania State University, University Park, Pennsylvania 16802-1401

Introduction

IN a typical trajectory optimization problem the time of flight or propellant consumed is minimized, resulting in significant changes in one or more orbital elements. In situations where an upper-stage engine has failed to burn completely or a solid motor remains unfired, the optimization task is reversed; the goal is to expend as much propellant as possible, sometimes with little or no change in the orbital elements. In such cases the payload is often not in the desired orbit, and mission planners might want to retrieve the payload using the space shuttle and subsequently relaunch it. Safety concerns might prohibit returning the payload and upper stage in the shuttle cargo bay with propellants still onboard. This problem was first encountered in 1984 with the Westar and Palapa B spacecraft, which failed to achieve their intended orbits. The solid-propellant apogee kick motors were fired to deplete the propellants, and the spacecraft were safely retrieved.

Generally, there are no means to vent liquid propellants, and they must be consumed by reigniting the engine. The space shuttle itself must use this method in the event that one of the orbital maneuvering system pods is leaking propellant; standard procedure calls for consuming this leaking propellant by firing the thruster out of plane, thus minimizing the changes in semimajor axis and eccentricity. In this Note it is assumed that the vehicle is presently in an orbit compatible with shuttle operations for satellite retrieval. Using only a single sustained burn, the thrust generated needs to be directed so as to minimize the changes in semimajor axis a, eccentricity e, and inclination i, ensuring that the new orbit would be accessible by a later shuttle mission.

Solution Method

This method employs a combination of extremal control laws, each of which extremizes one orbital element. These control laws will also be straightforward to implement. Spencer and Culp[1] used this approach to generate control laws that maximized rates of change for several orbital elements individually on a sequence of discrete burn-arcs. Kluever and Oleson[2] blended the extremal control laws to change simultaneously several orbital elements in a low-thrust problem.

This method requires control laws that each minimize a single orbital element's time derivative. For a satellite's retrieval, changes in its semimajor axis, eccentricity and inclination directly affect its accessibility by the shuttle. Changes in longitude of the ascending node, argument of periapsis, and time of periapsis passage are not as crucial because these can be accommodated through proper selection of shuttle launch time and subsequent orbital phasing.

Under the assumption of a spherical gravity field, satellite thrust results in the following rates for a, e, and i:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2a^2 v}{\mu} a_{\mathrm{TH}} \cos\theta \cos\sigma$$
$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{a_{\mathrm{TH}}}{v}[2(e + \cos f)\cos\theta + (r/a)\sin f \sin\theta]\cos\sigma$$
$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{r}{\sqrt{\mu a(1-e^2)}}\cos(\omega + f) a_{\mathrm{TH}} \sin\sigma \tag{1}$$

where v is the velocity, μ is the gravitational parameter, a_{TH} is the thrust acceleration (assumed constant in this Note), θ is the thrust angle (measured between the velocity vector and the thrust vector's projection onto the orbital plane), f is the true anomaly, ω is the argument of periapsis, and σ is the angle between the orbital plane and the thrust vector.

Zero rate in a requires the steering angle

$$\theta_a^* = \pm\pi/2 \tag{2}$$

while zero rate in e requires

$$\theta_e^* = \tan^{-1}\left[\frac{2(e+\cos f)(1+e\cos f)}{(e^2-1)\sin f}\right] \tag{3}$$

and zero rate in i requires

$$\sigma_i^* = 0 \tag{4}$$

Using the RTN (radial, transverse, orbit-normal) coordinate system, the unit vectors giving the corresponding thrust direction for minimum rates of change in a, e, and i, respectively, are

$$\hat{u}_a = \left[\sin\left(\theta_a^* + \gamma\right), \cos\left(\theta_a^* + \gamma\right), 0\right]^T$$
$$\hat{u}_e = \left[\sin\left(\theta_e^* + \gamma\right), \cos\left(\theta_e^* + \gamma\right), 0\right]^T, \qquad \hat{u}_i = [u_{i,R}, u_{i,T}, 0]^T \tag{5}$$

where γ is the flight-path angle (measured from the local horizontal plane to the velocity vector). The radial and transverse components of $\hat{u}_i$ are irrelevant. Blending the control laws consists of forming a linear combination of $\hat{u}_a$, $\hat{u}_e$, and $\hat{u}_i$ with time-varying proportions, then normalizing to generate a new unit vector. Because minimizing i requires no out-of-plane thrusting, using that extremal control would effectively remove σ as a degree of freedom. Therefore, $\hat{u}_i$ is modified to permit out-of-plane thrusting: $\hat{u}_i = [0, 0, 1]^T$. The weighting function for i directly controls the amount of out-of-plane thrusting. This will allow the optimizer to find a solution that modulates the inclination. The unit vector is

$$\hat{u} = \frac{G_a(t)\hat{u}_a + G_e(t)\hat{u}_e + G_i(t)\hat{u}_i}{\|G_a(t)\hat{u}_a + G_e(t)\hat{u}_e + G_i(t)\hat{u}_i\|} \tag{6}$$

where the $G_x(t)$ are the weighting functions (gains) for the steering laws. This unit vector defines the thrust direction in the RTN system. The optimization consists of determining the $G_x(t)$ for a, e, and i in discretized form.

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*Graduate Research Assistant, Department of Aerospace Engineering, 229 Hammond Building; txc179@psu.edu. Student Member AIAA.
†Professor of Aerospace Engineering, Department of Aerospacee Engineering, 229 Hammond Building; rgmelton@psu.edu. Associate Fellow AIAA.
‡Assistant Professor of Aerospace Engineering, Department of Aerospace Engineering, 229 Hammond Building; dbs9@psu.edu. Associate Fellow AIAA.

This is now a nonlinear programming problem, with the discretized values of $G_x(t)$ as the optimization variables. The sequential quadratic programming code NPSOL[3] was used to solve this nonlinear programming problem numerically. In this problem the objective function is

$$J = w_a(\Delta a/a_0)^2 + w_e(\Delta e/e_0)^2 + w_i(\Delta i/i_0)^2 \tag{7}$$

where w_a, w_e, and w_i are weighting factors associated with the changes in the elements Δa, Δe, and Δi, and a_0, e_0, and i_0, are the values prior to the burn. These changes were computed via fourth-order Runge–Kutta integration of the equations of motion

$$\ddot{r} - \dot{\alpha}^2 r = -\frac{\mu r}{(r^2+z^2)^{\frac{3}{2}}} + a_{\mathrm{TH}}\hat{u}_R, \qquad r\ddot{\alpha} + 2\dot{r}\dot{\alpha} = a_{\mathrm{TH}}\hat{u}_T$$

$$\ddot{z} = -\frac{\mu z}{(r^2+z^2)^{\frac{3}{2}}} + a_{\mathrm{TH}}\hat{u}_N \tag{8}$$

where r is the radial position; α is the angular position; z is the normal position; and $\hat{u}_R$, $\hat{u}_T$, and $\hat{u}_N$ are the components of $\hat{u}$ in the radial, transverse, and normal directions, respectively. The time step for the integration was 1 s.

Initial studies included the starting time of the burn as an optimization parameter; however, this introduced multiple local minima, degrading the performance of the optimizer. Detailed numerical analysis revealed regions of local optima for burn start times near periapsis and apoapsis.

Results

Several cases that represent shuttle accessible orbits were examined. In each case the thrust acceleration is 7 m/s^2, and the burn duration is 288 s. These values are based on the performance characteristics of the Centaur upper stage with a payload intended for a geostationary transfer orbit.[4] Because the thrust direction $\hat{u}$ depends only upon the relative proportions of $G_a(t)$, $G_e(t)$, and $G_i(t)$, the discretized values of $G_a(t)$ are set to 1; this reduces the number of optimization variables. In these examples $G_e(t)$ and $G_i(t)$ have each been discretized into 10 values spread evenly over the duration of the burn, with linear interpolation employed for intermediate time values.

The orbit chosen has an initial eccentricity of 0.003, a semimajor axis of 6620 km, and an inclination of 30 deg. Four cases were examined. Case 1 had no restrictions on the thrust direction. Cases 2 and 3 had a restriction placed on the thrust direction to ensure the spacecraft could physically accomplish the maneuver. The thrust direction for case 2 could not change by more than 1.5 deg/s, corresponding to a satellite capable of rapid slewing maneuvers, and for case 3 it could not change by more than 0.02 deg/s, typical of a satellite with low slew-rate capability. For the preceding cases all of the initial values for $G_e(t)$ and $G_i(t)$ were set to 0, exclusively maximizing the reduction in a as a starting condition. Case 4 examined the potential benefits of modulating the inclination: the first five gains for inclination were set to 1000, and the last five to -1000. In all four cases the weighting factors were chosen to emphasize a reduction in a, with values of $w_a = 100$, $w_e = 0.001$, and $w_i = 0.01$. For all cases the thruster burn was started at 2680 s into the orbit, which is apoapsis.

All four cases demonstrated acceptable performance with a, e, and i nearly unchanged from before the burn (Table 1). The thrust angles over the burn are found in Fig. 1. In case 1 the thrust is directed first at -90 deg, then abruptly switches to 90 deg, and subsequently switches twice more before the end of the burn; practical

Table 1 Numerical Results

Case	Δa, km	Δe	Δi, deg	Objective function
1	0.0025	7.57×10^{-8}	-7.31×10^{-6}	1.46×10^{-11}
2	-0.779	-2.20×10^{-4}	-0.353	8.13×10^{-6}
3	-6.47×10^{-5}	-8.72×10^{-9}	-8.13×10^{-6}	1.87×10^{-14}
4	-0.0031	-5.06×10^{-7}	-0.162	2.92×10^{-7}

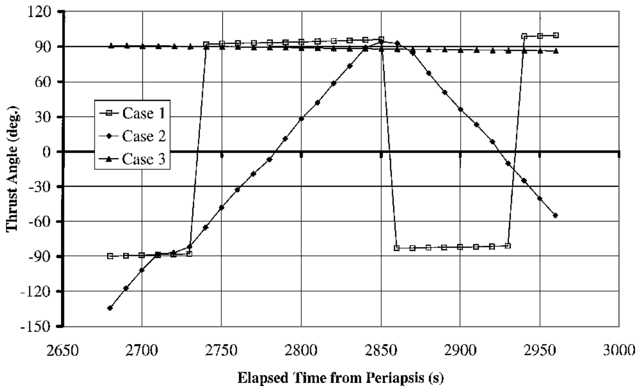

Fig. 1 Time history of thrust angle.

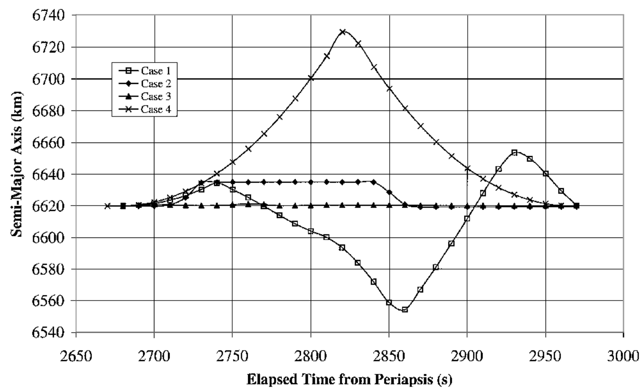

Fig. 2 Time history of semimajor axis.

constraints might prohibit such high slew rates. In case 2 the thrust angle varies between 90 and -90 deg smoothly because of the thrust angle restriction. This maneuver is not as effective and produces a significant change in a and i. The best solution occurs in case 3, when the thrust angle cannot change by more than 0.2 deg every 10 s.

This constraint drives the optimizer to find a solution that is superior to that of the unconstrained case 1 and serves to illustrate the multimodal nature of the objective function. When the results for case 3 are used as the initial values for an unconstrained case, the solution obtained is different from case 3 and case 1. The objective function for this case is 3.27×10^{-12}, which is a better performance than the original case 1, but still worse than case 3. Although the optimizer cannot be guaranteed to find the global optimum, the practical result in either case is no significant change in the critical orbital elements. The final solution found in case 4 consisted mainly of inclination modulation, with a small net change in inclination.

During the burn, a and e increase and then decrease for cases 1 and 2. The propellant is expended changing these parameters. In case 3 a and e change very little over the burn. At apoapsis the thrust angle for no change in a and e, Eqs. (2) and (3), is 90 deg. This is the basis for the solution to case 3. For all three cases there is little change in inclination. For case 4 all three elements are modulated to some degree, with most of the propellant expended changing the inclination from 30 to 22 deg and then back to 30 deg. Figure 2 shows the variation of semimajor axis over the burn for all of the cases. With regard to the less critical orbital elements, the net change in right ascension of the ascending node does not exceed 2.5 deg, whereas the argument of perigee change varies from 0.01 deg for case 3 to 103 deg for case 2.

Conclusions

Satellite retrieval might require expending unused propellant for safety reasons prior to the retrieval. Blended extremal controls that minimize changes in semimajor axis, eccentricity, and inclination during a single, constant-thrust burn can result in modest requirements for thrust steering. The method has been shown to produce acceptable changes in these orbital elements. The optimization program finds solutions that include very little out-of-plane thrusting.

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